Optimal Income Taxation: Recent Empirical Applications.

OPTIMAL INCOME TAXATION: RECENT EMPIRICAL APPLICATIONS

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1. *Introduction*

Many authors (Atkinson (1972), Tuomala (1984, 1990), Diamond (1998), Saez (2001) etc.) have explored the empirical and policy implications of the conditions established by Mirlees (1971) for the optimality of the personal income tax-transfer rule.

Two main focal points of interests have dominated these investigations: *i*) the profile of marginal tax rates; *ii*) the design of the mechanism of income support.

We can distinguish two «generations» of studies. The first (discussed in Section 2), from 1971 up to the late '90s, mainly consists of illustrative numerical exercises rather than of empirical applications. The optimal tax-transfer regime that most commonly emerges from these efforts is a negative income tax + a (almost) flat tax.

The second generation (Section 3) is characterized by a more definite focus on the policy implementation of the optimal taxation theory. These studies rely on extensions or reformulations of Mirrlees's model and attempt to establish a closer connection between theory and data or econometric estimates.

The present paper argues (Section 4) that both generations of studies suffer from taking for granted that the solution to the optimal taxation problem must be an analytical one (a «formula»), to be fed with numerical guesses or estimates. As a first consequence, the theoretical models must adopt very restrictive assumptions in order to generate analytical solutions. As a second

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consequence, the theoretical results are potentially inconsistent with the empirical estimates that are typically generated under very different (much less restrictive) assumptions.

We illustrate (Section 5) two exercises adopting a different approach (a third generation?), where restrictive theoretical assumptions are avoided and the solution is obtained computationally by iteratively simulating a very flexible microeconometric model.

2. *The First Generation*

We start by recalling the basic model introduced by Mirrlees (1971) – in a version with preferences separable in income and effort.

Agents – households – differ only in their market productivity w.

An agent with productivity w solves

$$\begin{aligned} &\max_{c,h} u(c,h) = U(c) - V(h) \\ &\text{c.v.} \\ &c = wh - T(wh) \end{aligned} \tag{2.1}$$

where
w = market productivity (e.g. wage rate)
c = net available income
h = hours of work (or, more generally, effort)
$T(z)$ = tax to be paid by an agent with income $z = wh$

The Social Planner solves

$$\begin{aligned} &\max_{c,h} \int_0^\infty W(u^w) f(w) dw \\ &\text{s.t.} \\ &\int_0^\infty T(z^w) f(w) dw = R \\ &u^w = \max_{c,h} u(c,h) \text{ c.v. } c = wh - T(wh) \end{aligned} \tag{2.2}$$

where

$$\begin{aligned} &z^w \equiv wh^w \\ &h^w = \arg\max_h u(c,h) \text{ c.v. } \mathrm{c} = wh - T(wh) \\ &f(w) \ = \ F_w(w) \ = \ \text{probability density function of } w \end{aligned}$$

The social planner knows the distribution $F(w)$ but not the individual values of w (or at least she is not willing – or allowed – to use them). The tax T only depends on wh (second-best solution).

By solving (2.2) we get:

$$(2.3) \quad \frac{T_z(z^w)}{1-T_z(z^w)} = \left[\frac{1+\varepsilon(w)}{\tilde{\varepsilon}(w)}\right] \times \left[\frac{(1-F(w))}{wf(w)}\right] \times \left[\frac{U_x^w \int_w^\infty (1-g^m)(1/U_x^m) f(m) dm}{1-F(w)}\right]$$

where $g^m \equiv \dfrac{W_u U_x^m}{\lambda}$

$e(w)$ = elasticity of the supply of h with respect to the net wage

$\tilde{\varepsilon}(w)$ = compensated elasticity of the supply of h with respect to the net wage

and λ is the multiplier associated to the public budget constraint (= social value of public funds), so that g^m is the social marginal value (relative to public funds) of a w-agent's income.

In expression (2.3), in general Y^w denotes a function Y evaluated at the optimal choice made by an agent with productivity w, Y_s^w denotes the first derivative of Y^w with respect to some variable s.

Expression (2.3) together with the public budget constraint also determines the amount of a lump-sum transfer (positive or negative).

Expressions like (2.3) since 1971 have been used by many authors in order to investigate the implications of the model with numerical illustrations. These exercises require specifying:

i) the utility function u;

ii) the productivity distribution function F;

iii) the Social Welfare Function W;

iv) the required amount of total net taxes R.

As a loose guide to deciding upon the above specifications, the authors adopt a «calibration» criterion, meaning that the chosen specification should conform to some degree of realism (for example it should generate a distribution of gross income that resembles a real one).

These exercises have been mostly disappointing for those – probably including Mirrlees himself – who expected an optimal profile of monotonically increasing marginal tax rates. In fact, the results tend to converge to the following optimal tax-transfer rule:

– a lump-sum (positive) transfer;

– very high (and declining) marginal tax rates on low incomes up to a break-even point (where gross income is equal to the transfer);

– constant or slightly increasing marginal tax rates beyond the break-even point.

If we translate the above scenario into a policy reform, by and large it corresponds to a Negative Income Tax + a (almost) Flat Tax (NIT + FT).

More or less egalitarian Social Welfare functions imply different level of the transfer and of the average tax but do not significantly change the marginal tax rates profile.

Up to recent years, most researchers working in the optimal taxation area tended to conclude that the above scenario was the definitive lesson to be drawn from Mirrlees's model. If one was looking for a model envisaging a progressive profile of increasing marginal tax rates, it had to be searched for outside that model.

Recent contributions by Tuomala (2006; 2008) however, convincingly argue that the results recalled above are very likely to be forced by too restrictive or too unrealistic specifications chosen for the numerical simulation. In particular, it appears that typical assumptions such as: a constant elasticity of labour supply, a constant (or missing) income effect, common utility specification such as the Cobb-Douglas, the CES, or the quasi-linear, and productivity distributions such as the log-normal, all tend to favour something close to the NIT + FT scenario.

Tuomala (2008), for example, presents a numerical illustration that adopts a much more flexible specification for the utility function, namely a quadratic form. As a first consequence, he gets a pattern of heterogeneous (with respect to income levels) elasticities and income effects that closely matches the empirical evidence. Moreover he obtains an optimal solution that envisages a lump-sum transfer and a profile of monotonically increasing marginal tax rates, i.e. something that resembles a Universal Basic Income + a Progressive Tax (UBI + PT).

3. *The Second Generation*

The numerical exercises of the First Generation have mainly an illustrative character and do not (or at least not explicitly) aim at realistically evaluating or recommending specific reforms inspired by the optimal taxation results.

More recently, a series of contributions have tried to establish a more direct connection between optimal taxation theory and the design of policy reforms. As a corollary of the more policy-oriented position, these contributions develop new versions of the basic Mirrlees's model and use not only simple «calibration» procedures, as was the case with the First Generation studies, but also real datasets and econometric estimates.

Saez (2001; 2002) presents reformulations of Mirrlees's model more directly interpretable in terms of empirically observable variables. In particular, Saez (2002) develops a discrete model that assigns a crucial role to the relative magnitude of the labour supply elasticities at the extensive and at the intensive margin. This framework turned out to be very influential in the last few years and therefore we illustrate it with more details.

There are $J+1$ types of jobs, each paying (in increasing order) z_o, z_l, ...z_J. Job «0» is non-participation o unemployment.

Net available income on job j is

$$c_j = z_j - T_j \tag{3.1}$$

where T_j is the tax paid at income level z_j.

Each agent is characterized by one of the potential incomes z_o, z_l, ...z and if she decides to work she is allocated to the corresponding job.

The agent of type j decides to work if $c_j \geq c_o$.

The extensive margin (or participation) elasticity is defined as:

$$\eta_j = \frac{c_j - c_0}{\pi_j} \frac{\partial \pi_j}{\partial (c_j - c_0)} \tag{3.2}$$

where π_j is the proportion of agents on job of type j.

Working agents can also move to a different job if income opportunities change, but the movements (for reasons implicit in the assumptions of the model) are limited to adjacent jobs (i.e. from job j to job $j-1$ or job $j+1$).

The intensive margin elasticity is defined as:

$$\xi_j = \frac{c_j - c_{j-1}}{\pi_j} \frac{\partial \pi_j}{\partial (c_j - c_{j-1})}\,. \tag{3.3}$$

Then it turns out that the optimal taxes satisfy:

$$\frac{T_j - T_{j-1}}{c_j - c_{j-1}} = \frac{1}{\xi_j} \frac{\sum_{k=j}^{J} \pi_k \left[1 - g_k - \eta_k \frac{T_k - T_0}{c_k - c_0} \right]}{\pi_j} \tag{3.4}$$

where g_k is the marginal social value (relative to the value of public funds) of income at job k.

It must be noted that in the model there are no income effects and choices at the intensive margin are restricted in a very special way. Despite these limitations the model is attractive for several reasons:

– it assigns a crucial and easily interpretable role to the two type of elasticities;

– it is simple to implement empirically;

– it seems to fit well into the popular framework that models labour supply as a discrete choice;

– differently from Mirrlees (1971), it allows for negative marginal tax rates: this may be the case if the participation elasticities are sufficiently large with respect to the intensive margin elasticities.

So far we know four applications of this model:

Blundell *et al.* 2006 (optimal taxation of single mothers in Germany and UK);

Haan *et al.* 2007 (optimal design of *children benefits* in Germany);

Immervoll *et al.* 2007 (evaluation of income maintenance policies in European countries);

Brewer *et al.* 2008 (report prepared for the Mirrlees Commission for tax reform in the UK).

In all the four applications the model tends to attest the superiority of mechanisms such as in-work benefits (i.e. rules involving negative marginal taxes, or subsidies on the wage rate) over alternatives like NIT or UBI.

It might be the case that the simplifications adopted by the model somehow force the result. Since the role of the two types of elasticities is so predominant and since empirically the participation elasticity tends to be much larger than the intensive-margin elasticity, maybe it is not so surprising that the optimal rule turns out to be one envisaging subsidies on low wages rather than transfers on low incomes.

Nonetheless it is important that the model adds a new scenario (in-work benefits) as an alternative to what appeared to be the inescapable consensus – i.e. the NIT + FT – in the first generation of studies.

4. *A critique*

The studies belonging to the First and Second generation have much in common as to the type of solution they aim at and as to the relationship between the theoretical solution and the empirical evidence. More precisely:

1) The researcher looks for an analytical solution to the optimal taxation problem, i.e. a «formula» that allows to compute the optimal taxes or marginal tax rates as function of exogenous variables and parameters.

2) The numerical simulations consist in calculating the analytical solution with exogenous variables and parameters assigned numerical values produced

by «educated guesses» (first generation) or econometric estimates (more often in the second generation).

As an example, consider expression (3.4). We might start by defining in some natural way the J+1 type of jobs (or income level). We should be able to observe the proportion π_o, π_l, ...π_j of agents occupying the various positions and the corresponding (for example average) income level. Then we might borrow some econometric estimates of the elasticities. At that point we would be in the position of computing the optimal taxes.

What's wrong with this way of proceeding?

The original sin resides in the idea of an analytical solution. Clearly an analytical solution is very useful to understand (and teach) the «grammar» of the problem. But when it comes to evaluating or designing actual tax-transfer rules, insisting on an analytical solution as a starting point has many undesirable implications.

1) First, in order to get an analytical solution we are forced to adopt many simplifying and restrictive assumptions.

2) Second, when we «feed» the formulas with empirical measures, we are very likely to face an inconsistency between the theoretical results and the empirical evidence, since the latter was typically generated under assumptions that are very different for those that made it possible obtaining the former.

The interesting paper by Blundell *et al.* (2006) represents however an example of this potential inconsistency. This study adopts the model of Saez (2002) and in particular it uses expression (3.4) by assigning numerical values to the elasticities derived from estimates obtained with a detailed microeconometric model. However, we have seen that Saez (2002) assumes there are no income effects and specifies a very special and limited representation of choices at the intensive margin. None of these assumptions are shared by the microeconometric model used to measure the elasticities. The microeconometric model is much more flexible, it accounts for income effects and implies very different responses at the intensive margin. So we might expect that the elasticities produced by the microeconometric model do not fit well into the framework on which expression (3.4) is based.

Modern microeconometric models (in this area we are talking of models of labour supply) are based on very general and flexible assumptions. They can accommodate many realistic features such as general structures of heterogeneous preferences, simultaneous decisions of household members, non-unitary mechanisms of household decisions, complicated (non-convex, non continuous, non-differentiable etc.) constraints and opportunity sets, multidimensional heterogeneity of both households and jobs, quantitative constraints etc. It is simply not feasible (at least so far) to obtain analytical solutions for the optimal taxation problem in such environments. Yet those features are very relevant and important especially in view of evaluating or designing reforms.

In the next section we illustrate and exemplify an alternative procedure, which consists in using a microeconometric model to obtain a computational solution of the optimal taxation problem. Analytical solutions might still remain useful for example in suggesting promising classes of tax-transfer systems that can then be more deeply investigated with the microeconometric model. The latter, which primarily simulates the agents' choices by utility maximization, is embedded into a global maximization algorithm that solves the social planner's problem, i.e. the maximization of a social welfare function subject to the public budget constraint. The philosophy inspiring this approach is similar to the one adopted since long ago in engineering and recently and successfully also in other applications of mechanism design (auctions, negotiation procedures, matching markets etc.: Roth (2002) provides a very inspired survey). The analytical solution is complemented by computational experiments that account for a host of realistic features that cannot be included in the theoretical model:

> Consider the design of suspension bridges. The simple theoretical model in which the only force is gravity, and beams are perfectly rigid, is elegant and general. But bridge design also concerns metallurgy and soil mechanics, and the sideways forces of water and wind. Many questions concerning these complications can't be answered analytically, but must be explored using physical or computational models. These complications, and how they interact with the parts of the physics captured by the simple model, are the domain of the engineering literature (Roth 2002).

5. *A Third Generation?*

In this section I illustrate two examples of the computational approach sketched at the end of Section 4. The first exercise (Aaberge and Colombino 2008) identifies the optimal tax-transfer rule within a class of piecewise-linear rules. The second exercise adopts a similar methodology in order to investigate the performance of various alternative mechanisms of income support in European countries (Colombino *et al.* 2008).

5.1. *An optimal tax-transfer rule*

In Aaberge and Colombino (2008) the optimal taxation problem is formulated as follows.

$$
\begin{aligned}
&\max_{\vartheta} W(V(c_1,h_1,j_1),V(c_2,h_2,j_2),...,V(c_N,h_N,j_N)) \\
&\text{s.t.} \\
(5.1)\qquad &(c_n,h_n,j_n)=\underset{(w,h,j)\in B_n}{\arg\max}\, U_n(c,h,j) \text{ s.t. } c=f(wh,I_n;\vartheta),\forall n \\
&\sum_{n=1}^{N}(w_n h_n + I_n - f(w_n h_n, I_n;\vartheta)) \geq R.
\end{aligned}
$$

Agent n can choose a «job» within an opportunity set B_n. Each job is defined by a wage rate w, hours of work h and other characteristics j (unobserved by the analyst). Given gross earnings wh and gross unearned income I, net available income is determined by a tax-transfer function $c=f(wh, I, \vartheta)$ defined up to a vector of parameters ϑ.

For the sake of simplicity, the problem is illustrated here as if the agents were single individuals, but in fact the exercise treats both singles and couples.

The class of tax-transfer rules considered is defined as follows:

$$
(5.2)\qquad c=\begin{cases}
Z+T \text{ if } Z\leq E \\
Z+T-\tau_1(Z-E) \text{ if } E<Z\leq Z_1 \\
Z+T-\tau_1(Z_1-E)-\tau_2(Z-Z_1) \text{ if } Z_1<Z\leq Z_2 \\
Z+T-\tau_1(Z_1-E)-\tau_2(Z_2-Z_1)-\tau_3(Z-Z_2) \text{ if } Z_2<Z\leq Z_3 \\
Z+T-\tau_1(Z_1-E)-\tau_2(Z_2-Z_1)-\tau_3(Z_3-Z_2)-\tau_4(Z-Z_3) \text{ if } Z_3<Z
\end{cases}
$$

where is income after tax, Z is the sum of gross market income (earnings plus capital income) and taxable public transfers, T is a tax-free public transfer (positive or negative), E is the exemption level, (τ_1, τ_2, τ_3, τ_4) are the marginal tax rates applied to the four brackets of income above the exemption level, Z_1 is the upper limit of the first bracket, Z_2 is the upper limit of the second bracket, Z_3 is the upper limit of the third bracket and T is a lump-sum that can be positive (i.e. a lump-sum transfer) or negative (i.e. a lump-sum tax). Thus, each particular tax rule is characterized by the nine parameters: $\rho=(E, \tau_1, \tau_2, \tau_3, \tau_4, Z_1, Z_2, Z_3, T)$. In the exercise presented hereafter the top marginal tax rate is constrained to be $\tau_4\leq 0.75$[1].

For any given tax-transfer rule (i.e. any given value of ϑ) the choices by the agents are simulated by a microeconometric model with 78 parameters

[1] This upper limit is imposed for the sake of realism, since it is the highest top marginal tax rate on personal income reached in Norway in the period 1980-2000.

estimated on 1994 Norwegian data. The model allows for a very flexible representation of heterogeneous preferences and opportunity sets, it covers both singles and couples, and it accounts for quantity constraints and is able to treat any tax-transfer rule however complex[2]. Note that it would be hopeless to look for analytical solutions of an optimal taxation problem in such an environment.

As an illustration of the implications of the model, especially as to heterogeneity of behaviour, Tables 1 and 2 report the wage elasticities and the income elasticities of labour supply. In particular: Table 1 shows the sharp decline of wage elasticity w.r.t. income levels (especially for married females); Table 2 shows that income elasticities are far from irrelevant, thus casting reservations on the frequently adopted assumptions of zero income effects (the same observation holds concerning cross-wage elasticities in Table 1). The same model, when estimated on Italian data, produces a very similar pattern of elasticities[3].

The choices made by the N agents results in N positions (c_1, h_1, j_1), (c_2, h_2, j_2),..., (c_N, h_N, j_N) which are then evaluated by the social planner according to a Social Welfare function

$$W(V(c_1, h_1, j_1), \mathrm{V}(c_2, h_2, j_2),..., V(c_N, h_N, j_N)).$$

The function V is a common utility function used to make the N positions interpersonally comparable.

The Social Planner's problem therefore consists of searching for the value of the parameters ρ that maximizes W subject to the following constraints:

1) the various positions (c_1, h_1, j_1),..., (c_N, h_N, j_N) result from utility-maximizing choices on the part of the agents (incentive-compatibility constraints);

2) the total net tax revenue must attain a given amount R (public budget constraint).

The social welfare function W is of the rank-dependent type[4]. Depending on the value assigned to an inequality-aversion parameter it can represent different criteria such as Utilitarian, Gini, Bonferroni etc.

The Social Planner's problem is solved by iteratively running the microeconometric model according to a global maximization algorithm that efficiently scans the tax-transfer parameter space.

[2] Besides Aaberge and Colombino (2008), other papers also illustrate previous versions of the model estimated on Italian data: Aaberge, Colombino and Strøm (1999; 2004) and Aaberge, Colombino, Strøm and Wennemo (2000).

[3] See for example Aaberge, Colombino and Wennemo (2002).

[4] See Aaberge and Colombino (2008) for more details.

TAB. 1. *Labour supply elasticities with respect to wage for single females, single males, married females and married males by deciles of household disposable income. Norway 1994*

Family status	Type of elasticity	Income decile under the 1994 tax system	Female elasticities		Male elasticities	
			Own wage elasticities	*Cross elasticities*	*Own wage elasticities*	*Cross elasticities*
Single females and males	Elasticity of the probability of participation	I	0.59		0.00	
		II	0.45		0.00	
		III-VIII	0.06		0.06	
		IX	0.00		0.00	
		X	0.00		0.00	
		All	0.12		0.04	
	Elasticity of the conditional expectation of total supply of hours	I	–0.17		0.77	
		II	–0.04		0.00	
		III-VIII	–0.08		–0.08	
		IX	–0.07		0.00	
		X	0.00		0.00	
		All	–0.09		–0.02	
	Elasticity of the unconditional expectation of total supply of hours	I	0.42		0.77	
		II	0.42		0.00	
		III-VIII	–0.02		–0.02	
		IX	–0.07		0.00	
		X	0.00		0.00	
		All	0.02		0.02	
Married/cohabitating females and males	Elasticity of the probability of participation	I	1.03	–0.28	0.90	–0.23
		II	0.35	–0.14	0.79	0.00
		III-VIII	0.14	–0.23	0.13	–0.10
		IX	0.12	–0.12	0.06	–0.06
		X	0.07	0.00	0.06	–0.19
		All	0.21	–0.19	0.23	–0.11
	Elasticity of the conditional expectation of total supply of hours	I	1.51	–0.01	0.87	0.11
		II	0.62	–0.53	0.38	–0.08
		III-VIII	0.27	–0.24	0.18	–0.14
		IX	0.08	–0.22	0.02	–0.09
		X	0.19	–0.10	–0.02	–0.23
		All	0.31	–0.25	0.16	–0.13
	Elasticity of the unconditional expectation of total supply of hours	I	2.54	–0.29	1.77	–0.12
		II	0.97	–0.67	1.17	–0.08
		III-VIII	0.41	–0.47	0.31	–0.24
		IX	0.20	–0.34	0.08	–0.14
		X	0.26	–0.10	0.05	–0.42
		All	0.52	–0.42	0.39	–0.23

Source: Aaberge and Colombino (2008).

TAB. 2. *Labour supply elasticities with respect to non-labour income for single females, single males, married females and married males by deciles of household disposable income. Norway 1994*

Family status		Income decile under the 1994 tax system	*Females*	*Males*
	Elasticity of the probability of participation	I	–0.59	0
		II	0	0
		III-VIII	–0.71	–0.12
		IX	–1.38	–0.33
		X	–1.33	–0.83
	Elasticity of the conditional expectation of total supply of hours	I	0.43	0
		II	0	0
		III-VIII	0.08	0.05
		IX	–0.21	0.05
		X	–0.51	–0.42
	Elasticity of the unconditional expectation of total supply of hours	I	–0.18	0
		II	0	0
		III-VIII	–0.63	–0.07
		IX	–1.56	–0.29
		X	–1.81	–1.22
Married/cohab. females and males	Elasticity of the probability of participation	I	0	0
		II	0	0.07
		III-VIII	–0.16	–0.17
		IX	–0.23	–0.46
		X	–0.81	–0.82
	Elasticity of the conditional expectation of total supply of hours	I	0	0
		II	–0.05	–0.08
		III-VIII	–0.05	–0.03
		IX	–0.14	–0.01
		X	–0.22	–0.32
	Elasticity of the unconditional expectation of total supply of hours	I	0	0
		II	–0.05	–0.01
		III-VIII	–0.21	–0.20
		IX	–0.37	–0.47
		X	–1.01	–1.11

Source: Aaberge and Colombino (2008).

Table 3 displays the optimal tax-transfer systems according to four different social welfare criteria, in decreasing degree of inequality aversion from Bonferroni to Utilitarian.

Figure 1 illustrates the three optimal tax-transfer rules and the current (1994) rule as mappings from gross income to net income.

We can observe:

a) Under any social welfare function, the marginal tax rates are continuously increasing for all levels of income. The lump-sum transfer turns out to be a tax (modest in the Utilitarian and Gini cases, more substantial in the Bonferroni case) In view of a practical implementation, the lump-sum tax might be approximated by a reduction of the (almost) universal transfers that

TAB. 3. *Optimal tax rules according to alternative social welfare criteria*. (τ_1 constrained to be ≤ 0.75). Norway 1994*

	Bonferroni	Gini	Utilitarian
τ_1	0.06	0.16	0.23
τ_2	0.30	0.26	0.28
τ_3	0.39	0.38	0.33
τ_4	0.75	0.75	0.75
T	–11,900	–6,000	–2,800
E	29,000	21,000	24,000
Z_1	120,000	130,000	210,000
Z_2	220,000	230,000	280,000
Z_3	720,000	710,000	740,000

* E, T, Z_1, Z_2 and Z_3 are measured in thousands of 1994 NOK (1 NOK ≅ 0.12 Euros).
Source: Aaberge and Colombino (2008).

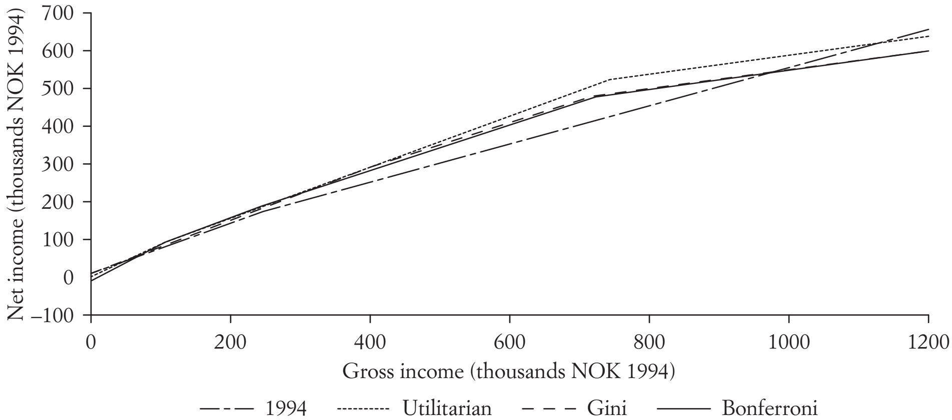

FIG. 1. Optimal tax-transfer rules versus current rule. Norway 1994.

characterize the current system (essentially family allowances and child benefits or by a tax on «unearned» income (wealth, houses, etc.). Altogether then the optimal tax-transfer rule might be seen as envisaging a universal transfer (a *demogrant*), a tax on wealth or property and a sequence of monotonically increasing marginal tax rates starting from 0 up to 75%. The above results are compatible with scenarios like UBI + PT or even In-Work Benefits mechanisms but not with scenarios like NIT + FT. This picture is in sharp contrast with most of the results obtained by the numerical exercises based on Mirrlees's optimal tax formulas. As we have seen in Section 2, the typical outcome of those exercises envisages a lump-sum transfer which is progressively taxed away by very high marginal tax rates on lower incomes (i.e. a negative income tax mechanism); beyond the income level where the transfer is exhausted, the marginal tax rates remains constant or slightly increasing. The papers by Tuomala (2006; 2008) also mentioned in Section 2 show however that those results might be forced by the restrictive assumptions typically made upon preferences, elasticities and the distribution of productivity (or wage rates). Interestingly, when Tuomala (2008) adopts a more flexible specification of the utility function he gets results that are qualitatively close to those in Aaberge and Colombino (2008).

b) The more egalitarian the criterion, the more progressive is the optimal rule. For example the optimal rule according to Bonferroni is more progressive than the optimal rule according to Gini, which in turn is more progressive than the optimal Utilitarian rule. However, even the Utilitarian criterion requires a very progressive rule, which suggests that the progressivity of the marginal tax rates responds to efficiency purposes as well as to equality purposes.

c) It turns out that all the optimal rules imply a higher income after tax for most levels of gross income with respect to the current system. In other words, the optimal rules are able to extract the same total tax revenue from a larger total gross income (i.e. applying a lower average tax rate). The result is due to a sufficiently high labour supply response estimated and accounted for by the model. The optimal rules induce (some of) the households to move to alternatives with longer hours and/or higher wages. Second, the optimal marginal tax rates applied to average or low and average income brackets are markedly lower than the ones implied by the current tax rule. This result provides a controversial perspective in view of the tax reforms implemented in many developed countries during the last decades. In most cases those reforms embodied the idea of improving efficiency and labour supply incentives through a lower average tax rate and lower marginal tax rates on higher incomes. Our optimal tax computations give support to the first part (lowering the average tax rate), much less to the second; on the contrary our results suggest that a lower average tax rate should be obtained by lowering the marginal tax rates particularly on low and average income brackets. Clearly the pattern of elasticities – sharply decreasing with respect to income – illustrated in Table 1 contributes to the profile of the optimal marginal tax rates.

5.2. *An evaluation of alternative basic income policies*

The second example is based on a preliminary report (Colombino *et al.* 2008) for a project in progress[5]. The approach is the same as in Aaberge and Colombino (2008) although the specification of the microeconometric model is somewhat simpler. The purpose is evaluating various different designs of basic income policies. Taxes are modelled in a very simple way, either a flat tax or a progressive tax with a constant progressivity index.

The results illustrated here are limited to four countries (Denmark, Italy, Portugal and United Kingdom). The exercise is devoted to evaluating and ranking the performance of various hypothetical reforms of the tax-transfer system.

Negative Income Tax + Flat Tax (NIT + FT):

This is a pure basic version of the widely discussed proposal originally and independently conceived by M. Friedman and J. Tobin. The rule is:

Net income = G if Gross Income <= G

Net income = G + $(1-t)$*(Gross Income − G) if Gross Income > G

where t is a constant marginal tax rate,

G = $aP\sigma$ = Minimum Guaranteed Income;

P = basic poverty line = (1/2) median household income in the sample;

a is a proportion (we simulate various versions with different values of a: 1, 0.75, 0.50 and 0.25),

σ is an equivalence scale that adjusts the basic poverty line according to the number of people (N) in the household.

The marginal tax rate t is endogenously determined by the simulation algorithm so that the net tax revenue is equal to the one collected under the current system.

Work Fare + Flat Tax (WF + FT):

This is similar to the NIT + FT, but the transfer to households with Gross Income < G is given only if either the husband or the wife (or both) work at least an average of H weekly hours[6]. In the simulation illustrated hereafter we set H = 20. This system, although based on a transfer rather than a wage subsidy, is close to some reforms recently introduced in the US and the UK

[5] *Reddito Minimo Garantito: un nodo cruciale nel disegno della politica sociale in Europa*. The research receives financial support from Compagnia di San Paolo and MIUR as a PRIN2006 project.

[6] See for example Fortin *et al.* (1993).

and currently discussed also in continental Europe (Earnings Tax Credit, In-Work Benefits etc.).

Participation Basic Income + Flat Tax (PBI + FT):

This is discussed among others by A. B. Atkinson (1996; 1998). Under this rule, every household receives a transfer equal to G (computed as above) irrespective of the Gross Income, provided either partner is working (any number of hours). Gross income is then taxed according to FT.

Universal Basic Income + Flat Tax (UBI + FT):

This is the basic version of the system discussed for example by Van Parijs (1995). Under this rule, every household receives a transfer equal to G (computed as above) irrespective of the Gross Income. Gross income is then taxed according to FT:

Net Income $= G + (1 - t)*$(Gross Income).

Negative Income Tax + Progressive Tax (NIT + PT):

Net income $= G + (\text{Gross income} - G)^{(1-\tau)}$ if Gross Income $> G$

where τ is a constant, and can be interpreted as an index of progressivity.

The parameter τ is endogenously determined by the simulation algorithm so that the net tax revenue is equal to the one collected under the current system.

Work Fare + Progressive Tax (WF + PT):

As with WF + FT, but we use PT instead of FT.

Participation Basic Income + Progressive Tax (PBI + PT):

As with PBI + FT, but we use PT instead of FT.

Universal Basic Income + Progressive Tax (UBI + PT):

As with UBI + FT, but we use PT instead of FT.

Table 4, 5 and 6 present an evaluation summary which focuses on four criteria, S(U), S(C), W(U) and W(C):

S(U) = Social Welfare (utility-based) = **Mean (U)*(1 – Gini (U)**), where Mean (U) and Gini (U) are respectively the mean and the Gini coefficient of the distribution of expected maximum utility levels attained by the households;

S(C) = Social Welfare (income-based) = **Mean (C)*(1 – Gini (C)**), where Mean (C) and Gini (C) are respectively the mean and the Gini coefficient of the distribution of net available income attained by the households;

W(U) = proportion of households whose expected maximum utility increases;

W(C) = proportion of households whose net available income increases.

Four each country and each criterion, we «grade» the reforms

with an «A» if it is the best one in that country according to that criterion;

with a «B» if it is the second best in that country according to that criterion;

with a «C» if it fares better than the current tax-transfer system in that country according to that criterion.

Table 4 shows the grades defined above for all the policies and all the countries. Overall, the most successful reforms are PBI and UBI, in particular in their progressive versions. PBI+PT and UBI+PT get 12 «A», 8 «B» and 75 «C». On the other hand, PBI+FT and UBI+FT get 3 «A», 11 «B» and 67 «C». Therefore there seem to be a clear indication of the superiority of non means-tested policies. A partial exception is represented by NIT in Italy, where it shows a performance almost comparable to that attained by PBI and UBI.

A second indication is that progressive systems seem to perform somewhat better than flat systems. We already noted that the progressive versions of PBI and UBI overall get higher grades than their non progressive versions. But this is true also of NIT. Progressive rules apply higher marginal tax rates on higher incomes and lower marginal tax rates on lower incomes (as compared to the flat rules). Members of households with higher income tend to show a lower elasticity of labour supply (w.r.t. wage). Therefore the progressive rules seem to exploit more efficiently the elasticity profile and induce the generation of a higher level of income. A third conclusion suggested by Table 4 is that for each country there are many reforms that would improve things according to at least one of the criteria. Italy appears to be the country the most amenable to a reform, in the sense that any type of basic income reform (in some version) would improve upon the current status. In this perspective, United Kingdom is somehow second after Italy, Portugal is third and last comes Denmark. Otherwise said, Denmark has, in relative terms, a very successful policy on income support and it is therefore difficult to improve upon it.

Immervoll *et al.* (2007) find that in-work benefits (close to our Workfare) dominate – on a social welfare basis – more universalistic transfer policies (close to our UBI or NIT). The picture emerging from our exercise is less clear-cut: as a matter of fact, a social welfare-based evaluation would suggest a slight superiority of the universalistic policies. The analysis of Im-

TAB. 4. *Summary evaluation of alternative basic income policies. All the policies*

		Denmark				Italy				Portugal				United Kingdom			
		S(U)	S(C)	W(U)	W(C)	S(U)	S(C)	W(U)	W(C)	S(U)	S(C)	W(U)	W(C)	S(U)	S(C)	W(U)	W(C)
NIT+FT	a = 1.00					A	C	C		C	C						
	a = 0.75					C		C								C	C
	a = 0.50							A	C								C
	a = 0.25							C	C								C
WF+FT	a = 1.00					C	C	C	C	C	C			C	C		
	a = 0.75						C	C	C						C	C	C
	a = 0.50						C	C	C								C
	a = 0.25								C								C
PBI+FT	a = 1.00	C	A	C	B	C	B	C	C		A			C	C	C	
	a = 0.75			C	C	C	C	C	C	B	C			B	B	C	
	a = 0.50			C	C	C	C	C	C	C	C			C	C	C	C
	a = 0.25						C	C	B	C					C	C	C
UBI+FT	a = 1.00	C	B	B	A	B		C	C	C	C			C	C	C	
	a = 0.75	C		C	C	B		C	C	C	C			B	C	C	
	a = 0.50			C	C			C	C	C	C			C	C	C	C
	a = 0.25							C	C						C	C	C
NIT+PT	a = 1.00					A	C	C		C	C			C	C	C	
	a = 0.75					C	C	C								C	C
	a = 0.50					C		B	C								C
	a = 0.25							C	C								C
WF+PT	a = 1.00					C	C	C	C	C	C			C	C		
	a = 0.75					C	C	C	C						C	C	C
	a = 0.50						C	C	C								C
	a = 0.25								C								C
PBI+PT	a = 1.00	B	C	A	C	C	A	C	C	C	B	C	C	C	C	C	
	a = 0.75	C		C	C	C	C	C	C	B	C	A	C	A	A	C	C
	a = 0.50			C		C	C	C	C	C	C	C	C	C	C	C	C
	a = 0.25						C	C	A			B	C		C	B	A
UBI+PT	a = 1.00	A	C	A	C	B		C		C	C	C	C	C	C	C	
	a = 0.75	C		C	C	B	C	C		A	C	C	C	A	C	C	
	a = 0.50						C	C	C	C	C	C	C	C	C	C	C
	a = 0.25						C	C	C			C	C	C	C	A	B

Source: Colombino *et al.* (2008).

mervoll *et al.* (2007) is based on theoretical optimal taxation results (Saez 2002) that require restrictive assumptions on preferences and choices (no income effects, no interaction among partners, little heterogeneity in behaviour), which might contribute to explaining the differences between their results and the ones illustrated here.

The above picture can change substantially if, besides the welfaristic criteria of Table 4, we also account for other criteria that might be relevant from the perspective of political sustainability. For example it might be argued that policy requiring «too high» top marginal tax rates could not be realistically considered. Table 5 excludes from the rankings the reforms that imply a top marginal tax rate higher than 55%. We choose this figure as a hypothetical politically feasible upper limit because it is close to the top marginal tax rate applied to personal incomes in European countries; in 2000, the four highest top effective marginal tax rates applied in Europe are 60.0% (Netherlands), 55.4% (Sweden), 54.3% Denmark and 53.8% (Germany)[7].

Other constraints to reform design and implementation might come from the implications on the choices or the conditions of specific segments of the population. For example the female participation rate is a matter of concern in the European political-economic debate. In Table 6 we further exclude from the grading the policies implying a reduction of female participation rate.

Table 6 suggests that in the countries with a relatively low female participation rate (Italy and Portugal) many welfare-improving policies do not survive to the application of the additional feasibility constraints: non means-tested policy like UBI or PBI appear to be too costly or have adverse incentives on labour supply; more selective policies such as WF or NIT are more likely to be feasible. On the other hand, in Denmark (the country with the highest female participation rate) all the welfare-improving policies survive. United Kingdom represents an intermediate case. Economic systems that have attained a high female participation rate are better equipped to implement universalistic basic income policies. Economic systems with low female participation rates tend instead to face a high price in terms of tax burden and supply disincentives.

[7] OECD tax database (http://www.oecd.org/ctp/taxdatabase).

TAB. 5. *Summary evaluation of alternative basic income policies, excluding policies implying a top marginal tax rate>55%*

		Denmark				Italy				Portugal				United Kingdom			
		S(U)	S(C)	W(U)	W(C)	S(U)	S(C)	W(U)	W(C)	S(U)	S(C)	W(U)	W(C)	S(U)	S(C)	W(U)	W(C)
NIT+FT	a=1.00									C	C						
	a=0.75					C		C								C	C
	a=0.50							A	C								C
	a=0.25							C	C								C
WF+FT	a=1.00					C	C	C	C	C	C			C	C		
	a=0.75						C	C	C						C	C	C
	a=0.50						C	C	C								C
	a=0.25								C								C
PBI+FT	a=1.00	C	A	C	B												
	a=0.75			C	C					B	C						
	a=0.50			C	C	C	C	C	C	C	C			C	C	C	C
	a=0.25						C	C	B	C					C	C	C
UBI+FT	a=1.00	C	B	B	A												
	a=0.75	C		C	C					C	C						
	a=0.50			C	C			C	C	C	C			C	C	C	C
	a=0.25							C	C						C	C	C
NIT+PT	a=1.00									C	C			C	C	C	
	a=0.75					C	C	C								C	C
	a=0.50					C		B	C								C
	a=0.25							C	C								C
WF+PT	a=1.00									C	C			C	C		
	a=0.75					C	C	C	C						C	C	C
	a=0.50						C	C	C								C
	a=0.25								C								C
PBI+PT	a=1.00	B	C	A	C												
	a=0.75	C		C	C					B	C	A	C				
	a=0.50			C		C	C	C	C	C	C	C	C	C	C	C	C
	a=0.25						C	C	A			B	C		C	B	A
UBI+PT	a=1.00	A	C	A	C												
	a=0.75	C		C	C												
	a=0.50						C	C	C	C	C	C	C	C	C	C	C
	a=0.25						C	C	C			C	C	C	C	A	B

Source: Colombino *et al.* (2008).

TAB. 6. *Summary evaluation of alternative basic income policies, excluding policies implying a top marginal tax rate>55% and/or a reduction in female participation rate*

		Denmark				Italy				Portugal				United Kingdom			
		S(U)	S(C)	W(U)	W(C)	S(U)	S(C)	W(U)	W(C)	S(U)	S(C)	W(U)	W(C)	S(U)	S(C)	W(U)	W(C)
NIT+FT	a=1.00																
	a=0.75																
	a=0.50																C
	a=0.25							C	C								C
WF+FT	a=1.00																
	a=0.75						C	C	C						C	C	C
	a=0.50						C	C	C								C
	a=0.25								C								C
PBI+FT	a=1.00	C	A	C	B												
	a=0.75			C	C												
	a=0.50			C	C												
	a=0.25						C	C	B	C					C	C	C
UBI+FT	a=1.00	C	B	B	A												
	a=0.75	C		C	C												
	a=0.50			C	C									C	C	C	C
	a=0.25														C	C	C
NIT+PT	a=1.00													C	C	C	
	a=0.75															C	C
	a=0.50																C
	a=0.25							C	C								C
WF+PT	a=1.00																
	a=0.75					C	C	C	C								
	a=0.50						C	C	C								C
	a=0.25								C								C
PBI+PT	a=1.00	B	C	A	C												
	a=0.75	C		C	C												
	a=0.50			C													
	a=0.25																
UBI+PT	a=1.00	A	C	A	C												
	a=0.75	C		C	C												
	a=0.50																
	a=0.25													C	C	A	B

Source: Colombino *et al.* (2008).

6. *Further perspectives for the Third Generation*

An objection to the approach exemplified in Section 5 might be that, indeed, it is example-dependent. A different dataset (a different country, the same country in a different year, a different demographic composition etc.) would likely produce different results. It seems that we have been able to avoid too restrictive assumptions but at the price of loosing generality. On the other hand, expressions (2.3) or (3.4) establish a precise and general relationship between optimal tax rates and characteristics of the economy such as the distribution of productivity, labour supply elasticity etc. Can we attain something similar within the computational approach? In principle the answer is affirmative. We can compute optimal taxes on many different economies, and then investigate the relationship between the characteristics of those economies and the corresponding optimal taxes. Certainly the objects to be put in relation are complex objects (structured clusters of variables and parameters), so the analysis would probably require a sort of generalized comparative statics: but appropriate techniques are becoming available for this purpose[8].

References

Aaberge R. - Colombino U. - Strøm S. (1999), Labour Supply in Italy: An Empirical Analysis of Joint Household Decisions, with Taxes and Quantity Constraints, in *Journal of Applied Econometrics*, 14, pp. 403-422.

Aaberge R. - Colombino U. - Wennemo T. (2002), *Heterogeneity in the elasticity of labor supply in Italy and some policy implications*, CHILD Working Paper, # 20/2002.

Aaberge R. - Colombino U. - Strøm S. (2004), Do More Equal Slices Shrink the Cake? An Empirical Investigation of Tax-Transfer Reform Proposals in Italy, in *Journal of Population Economics*, 17, pp. 767-785.

Aaberge R. - Colombino U. - Strøm S. - Wennemo T. (2000), Joint Labour Supply of Married Couples: Efficiency and Distribution Effects of Tax and Labour Market Reforms, in L. Mitton, H. Sutherland and M. Weeks (eds.), in *Micro-simulation Modelling for Policy Analysis: Challenges and Innovations*, Cambridge University Press.

Aaberge R. - Colombino U. (2008), *Designing Optimal Taxes with a Microeconometric Model of Household Labour Supply*, CHILD Working Paper, # 06/2008.

[8] I refer for example to the techniques pioneered by P. Milgrom and J. Roberts for the analysis of organizational structures, e.g. Milgrom and Roberts (1994).

Atkinson A.B. (1972), *Maximin and optimal income taxation*, Discussion Paper No. 47, University of Essex.
Atkinson A.B. (1996), The Case for a Participation Income, in *The Political Quarterly*, 67, pp. 67-70.
Atkinson A.B. (1998), *Poverty in Europe*, Oxford, Blackwell.
Blundell R. - M. Brewer - Haan P. - Shephard A. (2006), *Optimal Income Taxation of Lone Mothers: an Empirical Comparison for Germany and the UK*, mimeo.
Brewer M. - Saez M. - Shephard A. (2008), *Optimal Household Labor Income Tax and Transfer Programs: An Application to the UK*, http://www.ifs.org.uk/mirrlees-review.
Colombino U. - Locatelli M. - Narazani E. - O'Donoghue C. - Shima I. (2008), *Behavioural and Welfare Effects of Minimum Guaranteed Income Policies: A Simulation for European Countries*, CHILD Working Paper # 03/2008 (Also: EUROMOD Working Paper No. EM5/08).
Diamond P. (1998), Optimal Income Taxation: An Example with a U-Shaped pattern of Optimal Marginal Tax Rates, in *American Economic Review*, 88, pp. 83-95.
Fortin B. - Truchon M. - Beauséjour L. (1993), On Reforming the Welfare System: Workfare meets the Negative Income Tax, in *Journal of Public Economics*, 31, pp. 119-151.
Haan P. - Wrohlich K. (2007), *Optimal Taxation: The Design of Child Related Cash- and In-Kind-Benefits*, IZA Discussion Papers 3128.
Immervoll H. - Kleven H.J. - Kreiner C.T. - Saez E. (2007), Welfare Reforms in European Countries: A Microsimulation Analysis, in *Economic Journal*, 117, pp. 1-44.
Milgrom P. - Roberts J. (1994), Comparing Equilibria, in *American Economic Review*, 84(3), pp. 441-459.
Mirrlees J.A. (1971), An Exploration in the Theory of Optimal Income Taxation, in *Review of Economic Studies*, 38, pp. 175-208.
Roth A.E. (2002), The Economist as Engineer: Game Theory, Experimental Economics and Computation as Tools of Design Economics, in *Econometrica*, 70, pp. 1341-1378.
Saez E. (2001), Using Elasticities to Derive Optimal Income Tax Rates, in *Review of Economic Studies*, 68, pp. 205-229.
Saez E. (2002), Optimal Income Transfer Programs: Intensive versus Extensive Labour Supply Responses, in *Quarterly Journal of Economics*, 117, pp. 1039-1073.
Tuomala M. (1984), On the optimal income taxation: some further numerical results, in *Journal of Public Economics*, 23, pp. 351-66.
Tuomala M. (1990), *Optimal Income Tax and Redistribution*, Oxford, Clarendon Press.
Tuomala M. (2006), *On the shape of optimal non-linear income tax schedule*, Tampere Economic Working Papers, No. 49.
Tuomala M. (2008), *On optimal non-linear income taxation: numerical results revisited*, Mimeo, Department of Economics, University of Tampere.
Van Parijs P. (1995), *Real Freedom for All*, Oxford University Press.

Abstract:

In this paper we present a critical survey of two «generations» of studies addressing the empirical computation of optimal income taxes. The first generation, from 1971 up to the late ’90s, mainly consists of illustrative numerical exercises rather than of empirical applications. The optimal tax-transfer regime that most commonly emerges from these efforts is a negative income tax + a (almost) flat tax. The second generation is characterized by a more definite focus on policy implementations and, relying on extensions or reformulations of Mirrlees’s model, attempts to establish a closer connection between theory and data or econometric estimates. We argue that both generations of studies suffer from taking for granted that the solution to the optimal taxation problem must be an analytical one (a «formula»), to be fed with numerical guesses or estimates. As a first consequence, the theoretical models must adopt very restrictive assumptions in order to generate analytical solutions. As a second consequence, the theoretical results are potentially inconsistent with the empirical estimates that are typically generated under very different (much less restrictive) assumptions. A different approach would consist in avoiding restrictive theoretical assumptions and obtaining the solution computationally by iteratively simulating a microeconometric model. This approach is exemplified here with the computation of optima taxes in Norway and with the evaluation of alternative basic income mechanisms in some European countries.